Logarithmic derivatives

Given an exponential function in the form

$$y = \log_a \left[g(x)\right]$$

its derivative is

$$y' = \left[\frac{1}{g(x)\ln a}\right]\left[g'(x)\right]$$

In the case that our base $a = e$, we have a special $\log$ that's called $\ln$. In other words, $\log_e \left[g(x)\right] = \ln \left[g(x)\right]$. The natural log function

$$y = \ln \left[g(x)\right]$$

has a derivative of

$$y' = \left[\frac{1}{g(x)}\right]\left[g'(x)\right]$$

Let's try an example with $\log_a$.

Example

Find the derivative of the logarithmic function.

$$y = 6\log_8 \left(5x^4\right)$$

© Krista King Math

To find the derivative we need to apply the derivative formula for log functions.

$$y' = 6\left(\frac{1}{5x^4 \ln 8}\right)(20x^3)$$

$$y' = \frac{120x^3}{5x^4 \ln 8}$$

$$y' = \frac{24}{x \ln 8}$$

Now let's try an example with a natural log function.

Example

Find the derivative of the logarithmic function.

$$y = 5 \ln (2x^3)$$

To find the derivative we need to apply the derivative formula for natural logs.

$$y' = 5\left(\frac{1}{2x^3}\right)(6x^2)$$

$$y' = \frac{30x^2}{2x^3}$$

© Krista King Math

$$y' = \frac{15}{x}$$

Let's try one final example that's a little more complex.

Example

Find the derivative of the logarithmic function.

$$y = 9\ln\left(3x^7\right) + \left(4x^7\right)\left[\log_3\left(8x^2\right)\right] - 2x^{12}$$

We need to take the derivative one term at a time, applying the derivative formulas from the beginning of this section.

$$y' = 9\left(\frac{1}{3x^7}\right)\left(21x^6\right) + \left[\left(28x^6\right)\left(\log_3\left(8x^2\right)\right) + \left(4x^7\right)\left(\frac{1}{8x^2\ln 3}\right)(16x)\right] - 24x^{11}$$

$$y' = \frac{189x^6}{3x^7} + 28x^6\log_3\left(8x^2\right) + \frac{64x^8}{8x^2\ln 3} - 24x^{11}$$

$$y' = \frac{63}{x} + 28x^6\log_3\left(8x^2\right) + \frac{8x^6}{\ln 3} - 24x^{11}$$

© Krista King Math